Convection Diffusion in Hexagonal Rayleigh-Bénard Cell

OSAMU SANO

Department of Physics, Faculty of General Education, Tokyo University of Agriculture and Technology, Fuchu, Tokyo 183
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A study is made on the diffusion of particles in the thermal convection in a horizontal fluid layer, which is slightly above the critical Rayleigh number. In our experiment, in which almost periodic array of hexagonal cells is generated, a star-shaped spreading of neutrally buoyant particles is observed. Numerical simulation is performed, by taking a spatially periodic regular array of steady laminar hexagonal Rayleigh-Bénard cells as a basic flow, into which particular types of small disturbances are applied. Horizontal distribution of advected particles, particle paths and effective transport coefficient are investigated for various amplitudes, directions and frequencies of the perturbation. The characteristic pattern of impurity is reproduced in laminar hexagonal-cell-like flow with zero mean velocity, with enhanced effective transport coefficient which is proportional to the perturbation amplitude and which becomes maximum at some resonant frequency.

[convection diffusion, Rayleigh-Benard hexagonal cell, star-shaped impurity, spreading, resonant diffusion, environmental problem]

§1. Introduction

The effect of convective flows on the diffusion of matter is very large, with effective diffusion coefficient usually several orders of magnitude greater than the molecular diffusion coefficient. Thus it has great influence on many applications in chemical engineering and material processing which require uniform mixing or impurity distributions, as well as applications to environmental problems such as spreading of pollutants in the ocean, exhausted gases in the atmosphere, sludges in lakes and bays, and so forth.

It is well known that particles are transported quickly if they are placed in laminar flows with a certain non-zero mean velocity, or if they are placed in turbulent flows with or without mean velocity. On the other hand, it is only recently that enhancement of effective transport of particles in laminar fluid flows with zero mean velocity, like spatially periodic flows, has received considerable attention. Indeed many theoretical and experimental studies for the passive transport of two-dimensional Rayleigh-Bénard (RB) convection have been made, which elucidated the enhanced diffusion coefficient through the process described by chaotic advection. There are also many studies on mass transport in binary mixtures, in particular the existence of travelling waves, in which the coupling between temperature and concentration gradients plays an essential role. In these works, however, attention has been paid only to one-dimensional diffusive process.

If the basic flow has three-dimensional structure, the spreading of the diffusive matter is also expected to have spatial non-uniform distribution. Moreover, if some particular time-dependent small perturbation is applied to stationary basic flows, the effective transport will be greatly influenced by its amplitude and frequency, as well as the initial positions of the impurity source. As a consequence, this might lead to the appearance of highly contaminated region at a long distance from its source, or this might be used as a method of trapping the contaminants. In spite of the wide applications frequently encountered, the details of the effect of three-dimensionality have not been fully understood. In this paper, we briefly show our experimental results in hexagonal RB convection, and numerically simulate the mass transport in almost steady three-dimensional cellular flow to elucidate...
e characteristics of transport process underlying this particular type of RB convection.

### Experiment

We observed the thermal convection in a horizontal layer of silicon oil with density $\rho = 0.965$ g/cm$^3$, thermal expansion coefficient $\alpha = 9.5 \times 10^{-4}$ K$^{-1}$, thermal diffusivity $\kappa = 1 \times 10^{-3}$ cm$^2$/s, and kinematic viscosity $\nu = 1.00$ cm$^2$/s (at 25°C). Thus the Prandtl number $Pr = \nu / \kappa$ is about 910. The lower solid boundary was kept at a constant temperature $T_i = 80 \pm 1°C$, while the upper free boundary as kept at room temperature $T_e = 24 \pm 1°C$. The depth of the fluid $d = 0.30$ cm, so that the typical Rayleigh number $Ra = \alpha (\Delta T) gd^3/\nu \kappa$ of our observation is about 1300, which is slightly larger than the critical Rayleigh number 1100.6 for rigid-free boundary condition.

In the present experimental situation, the convection may also be influenced by the Marangoni effect which originates in the change of surface tension with temperature. An accurate estimation of the latter is not shown here owing to the difficulty in determining the rate of heat loss and its temperature dependence at the upper free surface. But our preliminary experiment, in which upper free surface was covered by rigid glass wall after the formation of convection cell, shows no drastic change of the pattern for a considerable time. Thus the buoyancy force is regarded as the main cause of our convection.

Convection cells were visualized either by shadowgraph or tracer method. In the latter case neutral buoyancy of the tracer particles is crucial for long time observation of the transport process. For this purpose we prepared pollens of pine with diameters of about 50 $\mu$m, each of which has two large empty bags advantageous to be carried by an air. These fine particles were immersed in the silicon oil for quite a long time, by which the pollen bags were filled with the same working fluid. Sedimentation of these particles was not noticeable during a few days' run of the experiment.

An almost regular array of hexagonal cells of typical horizontal size $l \sim 1$ cm, and typical velocity $v = 10^{-3}$ cm/s, with upward flow at the central part of the cell, was observed. The

![Fig. 1. Star-shaped particle spreading in hexagonal Rayleigh-Bénard convection; (a) photograph taken at time $t = 15$ min after the injection of particles, and (b) boundaries of a convection cell (broken lines).](image)

Péclet number $Pe = \nu l / D$, which is a measure of the relative importance of advection to diffusion in the transport, is about $10^5$ in our experiment, where the molecular diffusivity $D$ is estimated to be about $10^{-12}$ cm$^2$/s from the Stokes-Einstein relation. Thus the present transport process is predominantly caused by advection.

After the steady thermal convection was achieved, we injected tracer particles in the central part of the cell, and observed how they spread. Figure 1(a) is an example of photograph of the particles. Convection cells are also illustrated by dotted lines in Fig. 1(b). These figures clearly show the faster spread of tracers across the middle part of the side boundaries rather than the spread toward corners, which leads to a star-shaped expansion of impurities. The diffusion constant $D_{eff}$, determined by the Fick's law in which the particle flux from the central cell to neighboring ones is divided by the average concentration gradient, is about $4 \times 10^{-4}$ cm$^2$/s.

### §3. Numerical Simulation

#### 3.1 Model equation

We consider a Rayleigh-Bénard system of infinite lateral extent and thickness $d$. We choose the $z$ axis perpendicular to the fluid layer, and $x$-$y$ axes in the horizontal plane. We analyze the fluid motion on the basis of the Boussinesq equations, which are linearized by taking the steady heat conduction as an unper-